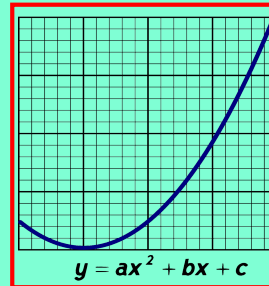


Math 125
Spring 2021
Lecture 9



Solve by graphing

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 4 \\ 4 & 0 \end{array} \quad \begin{array}{r|l} x & y \\ \hline 0 & -2 \\ 2 & 0 \end{array}$$

$$x + y = 4$$

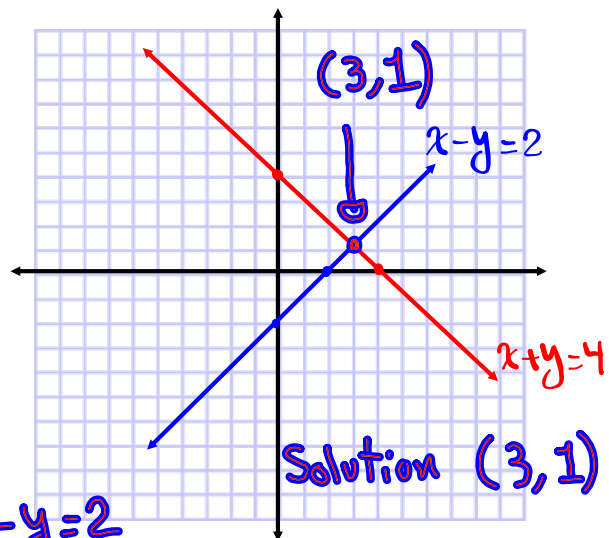
$$3 + 1 = 4$$

$$4 = 4 \checkmark$$

$$x - y = 2$$

$$3 - 1 = 2$$

$$2 = 2 \checkmark$$



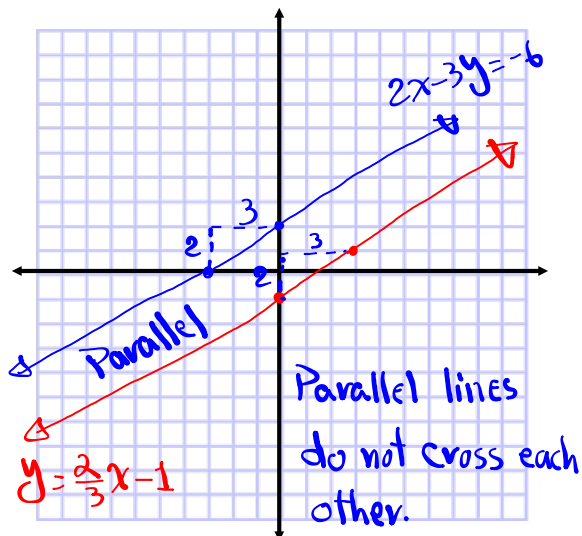
Solve by Graphing

$$\begin{cases} y = \frac{2}{3}x - 1 \\ 2x - 3y = -6 \end{cases}$$

x	y
0	2
-3	0

Y-Int (0, -1)

$$m = \frac{2}{3}$$



⇒ NO Solution

∅

Solve by graphing

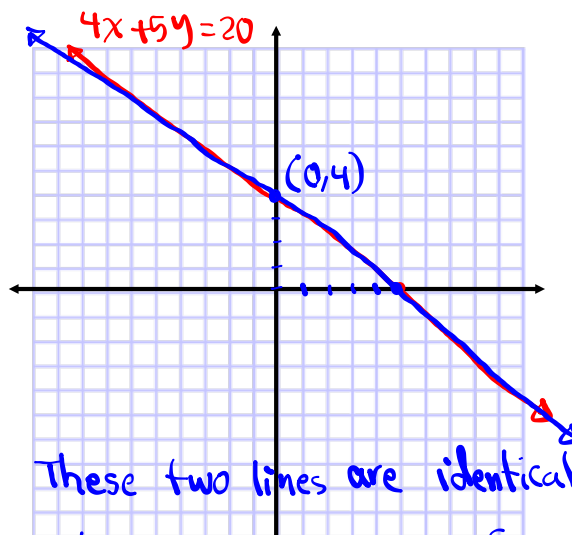
$$4x + 5y = 20$$

$$y = \frac{-4}{5}x + 4$$

x	y
0	4
5	0

Y-Int (0, 4)

$$\text{Slope } m = -\frac{4}{5}$$



These two lines are identical.

They have infinite # of Points in Common.

Infinite # of Solutions.

$$\left(x, \frac{-4}{5}x + 4\right) \quad x \text{ is a real \#.}$$

Solve by Substitution Method:

$$\begin{cases} 3x - 2y = 5 \\ y = x - 4 \end{cases} \quad \begin{aligned} & 3x - 2(x - 4) = 5 \\ & 3x - 2x + 8 = 5 \end{aligned}$$

$$x = 5 - 8$$

$$x = -3$$

$$y = -3 - 4$$

$$y = -7$$

Final Ans $\Rightarrow (-3, -7) \Rightarrow \{(-3, -7)\}$

Solve by Subs. method:

$$\begin{cases} 3x + y = 7 \\ 4x + y = 6 \end{cases} \quad \begin{aligned} & 3x + 6 - 4x = 7 \\ & -x + 6 = 7 \end{aligned}$$

$$-x = 7 - 6$$

$$-x = 1$$

$$x = -1$$

$$y = 6 - 4(-1)$$

$$y = 10$$

Final Ans: $(-1, 10) \Rightarrow$ Solution Set $\{(-1, 10)\}$

Solve by Subs. method:

$$\begin{cases} \frac{1}{2}x + \frac{2}{3}y = -4 \\ x - 2y = 8 \end{cases} \quad \begin{array}{l} \text{Clear fractions by using LCD} \\ \text{LCD} = 6 \\ 6 \cdot \frac{1}{2}x + 6 \cdot \frac{2}{3}y = 6(-4) \end{array}$$

$$\begin{cases} 3x + 4y = -24 \\ x - 2y = 8 \end{cases} \Rightarrow \text{Isolate } x: \Rightarrow x = 8 + 2y$$

$$3x + 4y = -24 \quad 3(8 + 2y) + 4y = -24$$

$$24 + 6y + 4y = -24$$

$$10y = -24 - 24$$

$$10y = -48$$

$$y = \frac{-48}{10}$$

$$y = -4.8$$

Recall

$$x = 8 + 2y$$

$$x = 8 + 2(-4.8)$$

$$= 8 - 9.6$$

$$x = -1.6$$

Final Ans:

$$(-1.6, -4.8)$$

Solve by Subs. method:

$$\begin{cases} 3x + 2y = 6 \\ y = \frac{-3}{2}x - 3 \end{cases}$$

Two parallel lines
Since y is isolated,
Plug it in other equation

$$3x + 2\left(\frac{-3}{2}x - 3\right) = 6$$

$$3x - 2 \cdot \frac{3}{2}x - 6 = 6$$

$$\cancel{3x} - \cancel{3x} - 6 = 6$$

$$-6 = 6$$

False

$$\Rightarrow \text{NO Solution}$$

$$\emptyset$$

Solve by Subs.

$$\begin{cases} 3x - 5y = 10 \\ y = \frac{3}{5}x - 2 \end{cases}$$

$$3x - 5\left(\frac{3}{5}x - 2\right) = 10$$

$$3x - \cancel{5} \cdot \frac{3}{\cancel{5}}x - 5(-2) = 10$$

$$\cancel{3x} - \cancel{3x} + 10 = 10$$

$$10 = 10$$

True

→ in finite # of Solutions

$$(x, y) =$$

$$\left(x, \frac{3}{5}x - 2\right)$$

x is a real #.

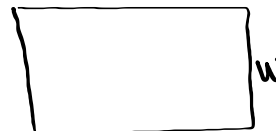
The perimeter of a rectangle is 20 ft.

Its length is 1 ft longer than twice its width.

Find its dimensions.

$$P = 2L + 2W$$

$$2L + 2W = 20$$



$$L = 2W + 1$$

$$\begin{cases} 2L + 2W = 20 \\ L = 2W + 1 \end{cases}$$

$$L = 2W + 1$$

using Subs. method

$$2(2W + 1) + 2W = 20$$

$$4W + 2 + 2W = 20$$

$$6W = 18$$

$$W = 3$$

$$L = 2W + 1$$

$$= 2(3) + 1$$

$$L = 7$$

7 ft by 3 ft

Jose is 9 yrs older than Maria.

Sum of their ages is 25 yrs.

use system of linear equations to find
their ages.

$$\begin{array}{l} x \rightarrow \text{Jose} \\ y \rightarrow \text{Maria} \end{array} \left\{ \begin{array}{l} x = y + 9 \\ x + y = 25 \\ y + 9 + y = 25 \end{array} \right. \rightarrow \begin{array}{l} 2y = 25 - 9 \\ 2y = 16 \\ \boxed{y = 8} \\ x = 8 + 9 \\ \boxed{x = 17} \end{array}$$

Jose is 17 yrs old,
and Maria is 8 yrs old.

D → Dimes

N → Nickels

Mike has 20 coins.

Mike has dimes & nickels only.

of dimes is 1 fewer than twice
of nickels. $D = 2N - 1$

1) How many of each does he have?

2) How much money does he have?

$$\begin{cases} \boxed{D + N = 20} \\ \boxed{D = 2N - 1} \end{cases} \quad \begin{array}{l} 2N - 1 + N = 20 \\ 3N = 21 \\ \boxed{N = 7} \end{array}$$

$$D = 2(7) - 1 \quad \boxed{D = 13}$$

Mike has 7 nickels &
13 dimes.

$$\begin{aligned} 7(5\text{¢}) + 13(10\text{¢}) &= \text{Total in ¢} \\ 35\text{¢} + 130\text{¢} &= \boxed{165\text{¢}} \\ &= \boxed{\$1.65} \end{aligned}$$

There were 52 people in a meeting.

of Females was 4 more than 3 times the # of males.

How many of each?

12 Males & 40 Females

$M \rightarrow$ Males, $F \rightarrow$ Females

$$\begin{cases} M + F = 52 \\ F = 3M + 4 \end{cases}$$

$$M + 3M + 4 = 52$$

$$4M = 48 \quad \boxed{M=12}$$

$$F = 3(12) + 4 \quad \boxed{F=40}$$

$$2 \begin{cases} x - y = 8 \\ 3x + 2y = 24 \end{cases}$$

Solve by addition/elimination method.

$$\begin{cases} 2x - 2y = 16 \\ 3x + 2y = 24 \end{cases}$$

$$\begin{array}{r} 2x - 2y = 16 \\ 3x + 2y = 24 \\ \hline 5x = 40 \end{array}$$

$$\boxed{x=8}$$

$$x - y = 8$$

$$8 - y = 8$$

$$\boxed{y=0}$$

Final Ans $(8, 0)$

Solution Set $\{(8, 0)\}$

Solve by addition/elimination method:

$$\begin{cases} 3x + 4y = 10 \\ -4(2x + y = 5) \end{cases} \Rightarrow \begin{cases} 3x + 4y = 10 \\ -8x - 4y = -20 \end{cases}$$

To eliminate y

$$\begin{array}{r} 3x + 4y = 10 \\ -8x - 4y = -20 \\ \hline -5x = -10 \end{array}$$

Final Ans: $(2, 1)$

Solution Set $\{(2, 1)\}$

$$\boxed{x=2}$$

$$\begin{array}{l} 2x + y = 5 \\ 2(2) + y = 5 \end{array} \quad \boxed{y=1}$$

Solve

$$\begin{cases} 4(5x - 3y = 16) \\ 3(3x + 4y = -2) \end{cases} \Rightarrow \begin{cases} 20x - 12y = 64 \\ 9x + 12y = -6 \end{cases}$$

$$\begin{array}{r} 20x - 12y = 64 \\ 9x + 12y = -6 \\ \hline 29x = 58 \end{array}$$

$$3(2) + 4y = -2$$

$$6 + 4y = -2$$

$$4y = -8 \Rightarrow \boxed{y = -2}$$

$$\boxed{x=2}$$

$$\boxed{(2, -2)}$$

Jose paid \$12 for 3 HB and 2 FF. ✓

Maria paid \$9.25 for 2 HB and 3 FF. ✓

Find the price for 1 HB. → eliminate FF.

$$\begin{cases} 3H + 2F = 12 \\ -2(2H + 3F = 9.25) \end{cases}$$

$$\begin{cases} 9H + 6F = 36 \\ -4H - 6F = -18.5 \end{cases}$$

$$5H = 17.5$$

$$H = \frac{17.5}{5} \quad H = 3.5$$

\$3.50

Mike has 11 coins.

N → Nickels

Quarters & Nickels only.

R → Quarters

He has \$2.15

How many of each?

$$\begin{cases} N + R = 11 \\ 5N + 25R = 215 \end{cases} \Rightarrow \begin{cases} N + R = 11 \\ N + 5R = 43 \end{cases} \Rightarrow \begin{cases} -5N - 5R = -55 \\ N + 5R = 43 \end{cases}$$

Divide by 5.

$$\begin{cases} 3 + 5R = 43 \\ 5R = 40 \\ R = 8 \end{cases}$$

$$-4N = -12$$

$$N = 3$$

3 Nickels &
8 Quarters

Allen needs 50 pounds of Coffee at \$2.60/lb.

He has unlimited supply of two brands of Coffee. One sells at \$2/lb. & other one at \$3/lb.

How many pounds of each should he mix to obtain what he needs?

$$\begin{array}{|c|} \hline \$2/\text{lb} \\ \hline x \\ \hline \end{array} + \begin{array}{|c|} \hline \$3/\text{lb} \\ \hline y \\ \hline \end{array} = \begin{array}{|c|} \hline \$2.60 \\ \hline 50 \\ \hline \end{array}$$

$$\begin{cases} x + y = 50 \\ 2x + 3y = 2.60(50) \end{cases} \xrightarrow{-2} \begin{cases} x + y = 50 \\ 2x + 3y = 130 \end{cases}$$

$$\begin{cases} -2x - 2y = -100 \\ 2x + 3y = 130 \end{cases} \quad \begin{cases} x + 3y = 50 \\ x = 20 \end{cases}$$

$$y = 30$$

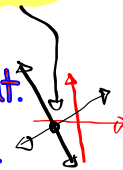
20 lb. @ \$2/lb. &
30 lb. @ \$3/lb.

Type of Systems & Equations:

When the system has **exactly one**
Solution:

Equations are independent.

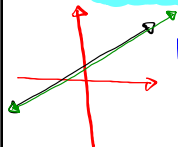
System is Consistent.



When the system has **infinite number of**
Solutions:

Equations are dependent.

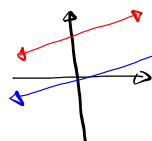
System is Consistent.



When the system has **no Solution:**

Equations are independent

System is inconsistent.



Simplify

$$\frac{x}{x^2-9} - \frac{x}{x^2+7x+12}$$

$$= \frac{\cancel{x}(x+4)}{(x+3)(x-3)\cancel{(x+4)}} - \frac{\cancel{x}(x-3)}{(x+3)\cancel{(x+4)}(x-3)}$$

$$= \frac{\underline{2(x+4) - 2(x-3)}}{(x+3)(x-3)(x+4)}$$

$$= \frac{x^2+4x - x^2+3x}{(x+3)(x-3)(x+4)} = \boxed{\frac{7x}{(x+3)(x-3)(x+4)}}$$

Solve

$$\frac{4}{x-6} - \frac{2}{x+6} = \frac{20}{x^2-36}$$

Hint: Use LCD to clear fractions

$$\text{LCD} = (x-6)(x+6)$$

$$\text{E.V.} \rightarrow \begin{array}{l} x-6 \neq 0 \\ x \neq 6 \end{array} \quad \begin{array}{l} x+6 \neq 0 \\ x \neq -6 \end{array}$$

$$\cancel{(x-6)}(x+6) \cdot \frac{4}{\cancel{x-6}} - \cancel{(x-6)}(x+6) \cdot \frac{2}{\cancel{x+6}} = \cancel{(x-6)}(x+6) \cdot \frac{20}{\cancel{x^2-36}}$$

$$4(x+6) - 2(x-6) = 20$$

$$4x + 24 - 2x + 12 = 20$$

$$2x + 36 = 20$$

$$2x = -16$$

$$\boxed{x = -8}$$

$$\{-8\}$$

Solve

$$-3x + 4 \geq 16$$

$$\text{OR } 2x - 7 > 3$$

$$-3x \geq 12$$

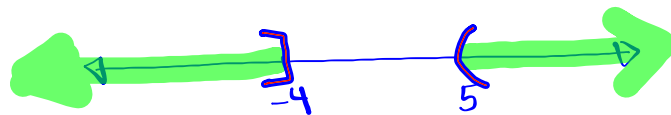
$$2x > 10$$

$$\frac{-3}{-3}x \leq \frac{12}{-3}$$

$$x > 5$$

$$x \leq -4$$

OR



$$\text{S.B.N. } \{x \mid x \leq -4 \text{ OR } x > 5\}$$

$$\text{I.N. } (-\infty, -4] \cup (5, \infty)$$

Solve

$$-2x + 3 < 7$$

$$\text{AND } 3x - 5 \leq 7$$

$$-2x < 7 - 3$$

$$3x \leq 7 + 5$$

$$-2x < 4$$

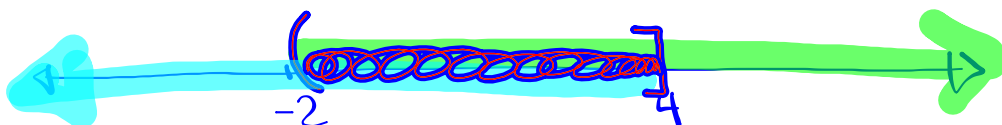
$$3x \leq 12$$

$$x > -2$$

overlap

AND

$$x \leq 4$$



$$\text{S.B.N. } \{x \mid -2 < x \leq 4\} \quad \text{I.N. } (-2, 4]$$

Class QZ 8

Solve $|2x-7| \geq 9$, graph, S.B.N., I.N.

You must use methods discussed in this class.